Vectorial Complex Ray Model for interaction of light with arbitrarily shaped smooth surface

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French project with 4 CNRS labs
Outline

• **Introduction**
  – Motivations
  – Limits of existing models
  – Consideration of new model

• **Vectorial Complex Ray Model**
  – Principle
  – Validation

• **Applications**
  – Rainbows and Airy theory
  – Caustics and cusp
  – Rainbow position of a liquid jet
  – Straight and twisted rainbows of a pendant drop

• **Conclusions and perspectives**
Introduction

- Optical metrology of particles in
  - Fluid mechanics
  - Combustion
  - Pharmaceutic
  - Microfluidic
  - ... ...

Spray in a motor

Spray of perfume

Microfluidic

Fuel atomization
Introduction

- Radar signal of a target (aero-plane ...).
- Wave propagation in the rain, raze ...
- Force on a non-spherical particle.
- ... ...

Interaction of light/wave with large irregular objects

- ...
Introduction

Theories and models

- **Rigorous theories:** Mie theory...
  - sphere, circular cylinder, small spheroid
  - limited to simple form

- **Numerical methods:** T-Matrix and DDA, FDTD, MoM, FMM, ...
  - Any shape
  - limited to small objects

- **Approximated models:**
  - Geometrical optics
    Often non-interference, partial solution or regular shape object, ...
  - Diffraction, GTD – geometrical theory of diffraction

New model to be developed
Introduction

Motivations

An example

- Understanding of the natural phenomena:
  - Fine structure of rainbow
  - Optical caustics, cusp and catastrophes

Rainbow scattering from spheroidal drops—an explanation of the hyperbolic umbilic foci


Can we predict the fine structure?
Motivations

An example

- Application to multiphase flow:
  - Fine structure of rainbow
  - Optical caustics, cusp and catastrophes

30 years ago
Marston $\rightarrow$ Nye

Today
1. Prediction of fine structure
2. Inversion

and

- Introduction

Motivations

An example
Introduction

Possible candidate?

- **Rigorous theories**: the particle shape must correspond to a coordinate system
- **Numerical methods**: size limited, very time consuming

✓ Ray models: precision to be improved

Key problem: lack of wave properties

Our strategy:

**Extension of ray model**

- Inclusion of wave front curvature
- Interference between all the rays.
- Diffraction.
Introduction

Our model
geometrical optics + wave properties

- Theoretical research:
  1. first step:
     Vectorial Complex Ray Model (VCRM):
     = Ray tracing + wave front curvature
  2. second step:
     - VCRM + interference + diffraction → Ray theory of wave
     - Results et applications:
       - VCRM validated by a rigorous numerical method (Yang et al, JQSRT, 2015),
       - VCRM validated by experiment for oblate particle (Onofri et al, OpEx, 2015)
       - Characterization of a water drop (ILASS 2016 Brighton).
       - 3D scattering by a non-spherical particle (CFTL 2016, Toulouse)
Vectorial Complex Ray Model (VCRM)

Geometrical optics + wave form

- Vectorial Complex Ray Model – new

✓ 5 properties of a ray:

1. direction,
2. amplitude,
3. phase,
4. polarization

New 5. Wave front curvature

✓ Advantages:
  • Objects of any shape with smooth surface,
  • Incident wave of any form,
  • Sufficiently precise – scattering in all directions,
  • All scattering properties of the objet.

For details:
- http://www.amocops.eu
Vectorial Complex Ray Model

• Fundamental laws
  1. Wave front equation:
     \[ (k_n^i - k_n^t)C = k' \Theta^i T Q' \Theta' - k \Theta^T Q \Theta \]
  2. Law of Snell-Descartes in vectors:
     \[ k^i_r = k^t_r \]

• Amplitude:
  \[ A = \sqrt{D \varepsilon} \]

• Phase:
  \[ \Phi = \Phi_{inc} + \Phi_{fl} + \Phi_{path} + \Phi_{\varepsilon} \]

• Total field:
  \[ E = S_{diff} + \sum_{i=1}^{N} S_i \]

Divergence factor:
\[ D = R_{11} R_{21} \frac{R_{12} R_{22}}{R_{12} R_{22}} \ldots \frac{R_{1q} R_{2q}}{R_{1q} R_{2q}} \]

Fresnel coefficients: \[ \varepsilon \]
\[ \tilde{r}_\perp = \frac{k_n - \tilde{k}_n}{k_n + \tilde{k}_n}, \quad \tilde{r}_\parallel = \frac{\tilde{m}^2 k_n - \tilde{k}_n}{\tilde{m}^2 k_n + \tilde{k}_n} \]

All expressed in wave vector components.
Vectorial Complex Ray Model

• Special case of the wave front equation:

When the rays remain in the same plane – a main direction of the wave front and particle surface:
  - Spherical particle
  - Infinite cylinder at normal incidence
  - Ellipsoidal particle in the symmetric plane.

- Curvature matrix:
  \[ C = \begin{pmatrix} \frac{1}{\rho_1} & 0 \\ 0 & \frac{1}{\rho_2} \end{pmatrix} \]

- Wave front equation:
  \[
  \frac{k_n^2}{k'R_1'} = \frac{k_n^2}{kR_1} + \frac{k_n' - k_n}{\rho_1} \\
  \frac{k'}{R_2'} = \frac{k}{R_2} + \frac{k_n' - k_n}{\rho_2}
  \]
Validation of the model

Theoretical Validation – divergence factor

• Sphere
  – Reflection: \( R_1 = -\frac{a \cos \alpha}{2} \) \( R_2 = -\frac{2}{a \cos \alpha} \) \( \rightarrow D = \frac{1}{4} \)
  – Refraction \( p = 1 \):
    After 1\(^{\text{st}}\) refraction: \( R'_{11} = -\frac{am \cos^2 \beta}{m \cos \beta - \cos \alpha} \) \( R'_{12} = -\frac{am}{m \cos \beta - \cos \alpha} \)
    After 2\(^{\text{nd}}\) refraction:
    \( R'_{21} = \frac{m \cos \beta - 2 \cos \alpha}{2(m \cos \beta - \cos \alpha)} \cos \alpha \) \( R'_{22} = \frac{2a \cos \beta(m \cos \beta - \cos \alpha) - m}{2(m \cos \beta - \cos \alpha)(\sin \alpha \sin \beta - \sin \alpha \sin \beta)} \)

  Divergence factor: \( D = \frac{m \sin(2\alpha) \cos \beta}{4 \sin(2(\beta - \alpha))(\cos \alpha - m \cos \beta)} \)
  Identical to the classical one.

• Cylinder: \( R_2 = \infty \)
  – Reflection:
  – Refraction \( p=1 \):

\[ D = \frac{a \cos \alpha}{2} \]
\[ D = \frac{m \cos \alpha \cos \beta}{2(\cos \alpha - m \cos \beta)} \]
Validation of the model

Comparison between VCRM, GO and LMT

Scattering diagrams by LMT, GO and VCRM for an infinite circular cylinder:
- refractive index: $m = 1.33$,
- radius $a = 50 \mu m$
- wavelength $\lambda = 0.6328 \mu m$.

Scattering diagrams by LMT, GO and VCRM for an infinite circular cylinder:
- refractive index: $m = 1.33$,
- radius $a = 5 \mu m$
- wavelength $\lambda = 0.6328 \mu m$. 
Validation of the model

Comparison between VCRM and GLMT

Sphere:
\( a = 30 \, \mu \text{m}, \)
\( m = 1.333 \)

Gaussian beam:
\( \lambda = 0.6328 \, \mu \text{m} \)
\( w_0 = 10 \, \mu \text{m} \)

Scattering diagrams calculated by VCRM and GLMT for a spherical particle of radius \( a = 30 \, \mu \text{m} \), refractive index \( m = 1.333 \) illuminated by a Gaussian beam of wavelength \( \lambda = 0.6328 \, \mu \text{m} \) and beam waist radius \( w_0 = 10 \, \mu \text{m} \).
Validation of the model

Comparison between VCRM and MLFMA

MLFMA:
Multilevel
Fast
Multipole
Algorithm

Comparison of scattering diagrams computed by VCRM and MLFMA for a prolate spheroid ($\kappa_1 = 1.0$, $\kappa_2 = 1.2$) and $m = 1.33$) illuminated by the plane wave of wavelength 0.785 µm with incident angle of $30^\circ$. The volume of the prolate is equal to that of a sphere of a radius 30 µm.

Figure 8 in
Yang et al JQRST 2015

MLFMA: Multilevel Fast Multipole Algorithm
Validation of the model

Experimental validation

- Experimental set-up

Figure 3 in Onofri et al, Opt. Exp. 2015
Validation of the model

Experimental validation

- Comparison of the results

Comparison of VCRM and experimental normalized equatorial scattering diagrams for the droplets of 3 different aspect ratios. From (a) to (c), the droplet's aspect ratio $b/a$ increases and refractive index decreases when the amplitude of the acoustic field is reduced.

Figure 3 in Onofri et al, Opt. Exp. 2015
The Colors of bows are due to the refractive index.

But the intensity tends to infinity!!
Airy theory revisited

Airy theory (1838)

- Phase difference is:
  \[ \Delta \Psi = \frac{k \nu v^3}{3a^2} \]
- Amplitude is constant for all emergent rays
- Amplitude of scattered field in \( q \) direction:
  \[ \int_{-\infty}^{\infty} e^{-ikv(\theta - \theta_0)} + ikv^3/3a^2 \, dv. \]

The version of van de Hulst (1957) permits to predict the profile,

- But the absolute/relative intensity?
Airy theory revisited

R. Wang and van de Hulst

Wang, van de Hulst et Lock: same principle but with amplitude correction:

\[
E_{\text{Airy}}^p(\theta) = x \left( \frac{2\pi \sin \theta_i^R}{\sin \theta_i^R} \right)^{1/2} \frac{x^{1/6}}{h^{1/3}} T^{21}(\theta_i^R) \times \left[ R^{11}(\theta_i^R) \right]^{p-1} T^{12}(\theta_i^R) \times \\
Ai\left(\frac{-x^{2/3} \Delta}{h^{1/3}}\right) \exp(2\pi i l_i^R/\lambda) \exp \left[ (ix\Delta) \left( \frac{p^2-n^2}{p^2-1} \right)^{1/2} \right]
\]

VCRM predicts better Airy bows than Airy theory!

Can anything new be said about the rainbow? Yes. The insight that this phenomenon arises from the play of light in a single spherical drop is 7 centuries old, the full geometrical optics theory of Descartes 3-1/2 centuries, and its modification by Airy to take account of diffraction just 1-1/2 centuries. Exactly a century ago
First order rainbow position of a spheroid with arbitrary ellipticity

The limit of Moebius model (1910 for $K \approx 1$ only) is surpassed.
Vectorial Complex Ray Model

• Application to ellipsoidal particle

The intensity ratio of the first bows depends on the aspect ratio.
Application to fluid mechanics (liquid jet)

- Experimental set up
Application to fluid mechanics (liquid jet)

- Comparison btw experimental and theoretical results

Profile of liquid jet

Profile of liquid jet

Relative height

Simulated rainbow positions.

measured rainbow positions.

Simulated rainbow positions.
Application to fluid mechanics (water drop)

- Experimental set-up

![Diagram of experiment setup](image-url)

1. Camera A
2. Motorized rotation platform
3. Suspended droplet
4. Camera B
5. Screen
6. Laser
7. Lens 1
8. Lens 2
9. 32.52 mm
10. 55 mm
11. 58 mm
Application to fluid mechanics (water drop)

- Diagrammes de diffusion expérimentaux *dans les angles arcs-en-ciel*

**First rainbows:**
1. Intensity decreases.
2. Intensity can be smaller than second rainbow.

**Second rainbows:**
1. Second rainbow is deformed
2. Deformation depends on the form of the droplet.
Application to fluid mechanics (water drop)

- Description of the geometry of the droplet

For scattering in the horizontal plane:

1. Simplified to an ellipsoid for the study of the intensity ratio of rainbows.

For 3D scattering diagram:

2. Described by a polynomial function for 3D simulation of full scattering diagram.

\[ r(\theta) = a_0 + \sum_{i=2}^{10} a_i \theta^i \]
Application to fluid mechanics (water drop)

- **Intensity ratio of the two rainbows (1)**

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</tbody>
</table>

**Conclusion:** The intensity ratio of the two rainbows decreases with the ellipticity of the drop.
Application to fluid mechanics (water drop)

• Diffusion par une goutte d’eau pendante

\[ r(\theta) = a_0 + \sum_{i=2}^{10} a_i \theta^i \]

Image expérimentale

Simulation numérique
Application to fluid mechanics (water drop)

- Diffusion par une goutte d’eau pendante

\[ r(\theta) = a_0 + \sum_{i=2}^{10} a_i \theta^i \]

Simulation numérique

Image expérimentale
Application to fluid mechanics (water drop)

- Diffusion par une goutte d’eau pendante

Zone d’éclairage

Image de la goutte 1

Image de la goutte 2

Image de diffusion 1

Image de diffusion 2
Application to fluid mechanics (water drop)

- Diffusion par une goutte d’eau pendante

Composition des images de la goutte et diagramme de diffusion
Conclusions and perspectives

• Conclusions:
  o VCRM has been developed and validated numerically and experimentally,
  o VCRM has been applied in the characterization of fluid mechanics,
  o Certain wave effects have been taken into account.

\[ VCRM_{T(RW)} \text{ is very promising for the scattering of larges non-spherical particles.} \]

• Perspectives:
  – Accomplishment of 3D case (analytical and numerical particles)
  – Interferences and diffraction,
  – Scattering of shaped beam,
  – Applications to the measurement of multiphase flows.
Software

- Free Software and code available:
  - VCRMEll2D with manual downloadable from:
    http://amocops.univ-rouen.fr/en/content/download
  - Fortran code to be requested to
    Fang.ren@coria.fr
Acknowledgements

• This work is mainly supported by French Research Angence for 4 CNRS research institutes:

[Images of logos]

• Link of the project AMO-COPS: [www.amocops.eu]

• Main collaborations:
  
  – Xidian university: Prof. X. Han, R. Zhong, Q. Duan, ...
  
  – Beijing Institute of Technology: Prof. X. Sheng, M. Yang, ...
  
  – Texas A&M University: Prof. P. Yang, B. Sun, G. Kattawar, ...
Thank you for your attention